A hydrodynamical description for transport in the strange metal phase of cuprates

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Based on work with
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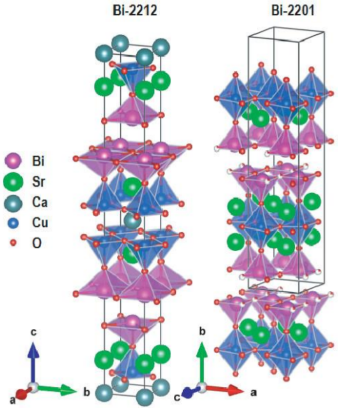
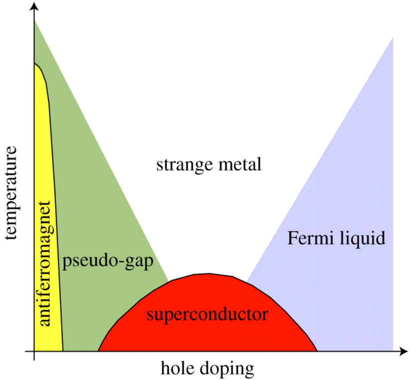
Cuprates

- Layers of CuO_2 planes bounded by rare earths
- Superconductivity and the most part of exotic properties happen on the CuO_2 plane $\rightarrow$ 2D materials
- Universal properties despite many different compounds
- Among High-T_c superconductors Bi-2201 has a relatively low critical temperature even at optimal doping $\Rightarrow$ ideal to test low T properties of the normal phase

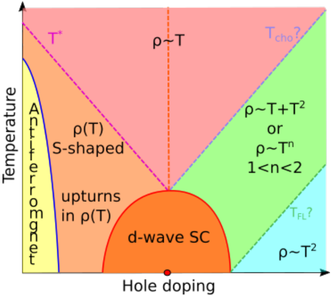


Cuprates phase diagram



- Cuprates have almost the same Temperature vs doping (concentration of rare earth) phase diagram, characterized by many intertwined phases appearing at the same time.

Phase diagram, QCP and scaling laws



- QCP is supposed to affect the properties of the strange metal phase:
  - transport coefficients should assume simple scaling laws
  - *Strong coupling:* no well defined quasi-particles.

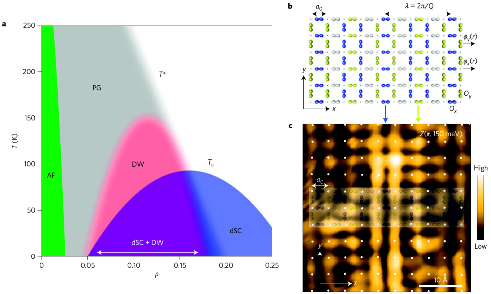
The Resistivity and Hall angle issue

- In normal Fermi liquid (magnetic field perpendicular to CuO_2 planes)
$$\rho_{xx} \sim T^2 \ , \qquad \cot\theta_H = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2$$
- In most of the cuprates
$$\rho_{xx} \sim T \ , \qquad \cot\theta_H = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2$$
- Actually in Bi-2201 is known that $\cot\theta_H \sim T^{1.5}$

Other transport coefficients are less known

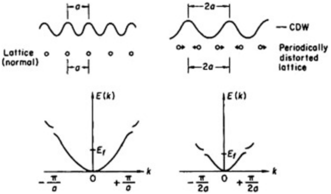
- Some of them are just dominated by lattice vibration
  - κ_{xx} has an 80 % of lattice phonon contribution
- Transverse transport coefficients are independent of phonons contribution (typically very small signal)
  - The Nernst coefficient N ([Wang, 2006] for a review)
  - The thermal Hall conductivity κ_{xy} (measured in LSCO [Grissonnanche, 2019] and in YBCO [Zhang, 2000][Matusiak, 2009])
  - Magnetoresistance typically B^2 suppressed

More orderings discovered recently



- Charge-density wave (CDW) order appears to be a ubiquitous feature of cuprate superconductors.
- Our material, $Bi_2Sr_2CuO_6$:
  - 2D CDW confirmed (by X-ray diffraction) to extend to optimal and over-doped region [Peng 2018],
  - low critical temperature ($T_c \sim 10 - 33$ K).

Charge density wave order



- **What are charge density waves?**
  - Peierls (1955) suggested periodic distortion of 1D lattice can lower total energy.
  - Start with first Brillouin zone $k = \pm\pi/a$ half filled.
  - CDW distortion $\rightarrow$ new superlattice of spacing $2a$. New first Brillouin zone band gap at $k = \pm\pi/2a$.
  - Gain in creating energy gaps can overcome loss of lattice distortion.
- Incommensurate CDW $\rightarrow$ broken translation invariance.

CDW and pinning

As soon as the translation SB is pseudo-spontaneous (Goldstone Bosons have a small mass) the AC conductivity can have an off-axes peak [Fukuyama-Lee-Rice '78,Delacretaz 2017]

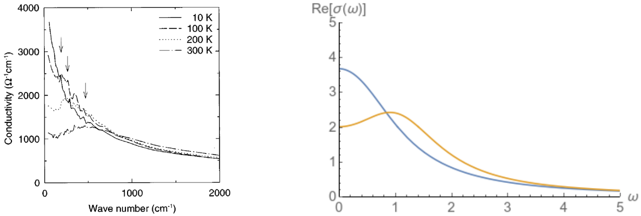


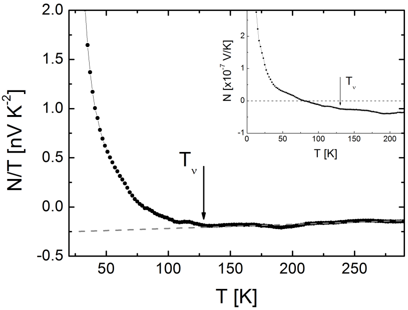
Figure: Experimental BiSCO conductivity from [Tsvetkov 1997]

$$\sigma(\omega) = \sigma_0 + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}$$

- for $\omega_0^2 > \Omega^3/(\Gamma + 2\Omega)$ there is an off-axes peak
- can the Drude to off axes peak originate from the same mechanism?

CDW not only affects the conductivity

- Usually the enhancement in the Nernst effect at low T was attributed to fluctuating superconductivity
- [Cyr-Choinière 2009] found a relation between T_{CDW} and the enhancement temperature



- T_ν is the temperature at which one recovers a Fermi Liquid expectation ($T_\nu \sim 2T_{CDW}$)
- CDW affects the Nernst signal also at fluctuating level

Where do we stand?

- Can one mechanism takes into account consistently all the thermo-electric transport coefficients?
- Many intertwined phases $\Rightarrow$ difficult to uncover
- We need a metallic behavior
- Strange metals are strongly coupled by nature

Hydrodynamics might come to help

Hydrodynamics as an EFT

- At large length and time scales, only a small number of DOFs survive to become hydrodynamic modes
  - If no spontaneously broken symmetries: (almost)-conserved currents.
- EOMS are determined by symmetries. Eg in a the relativistic charged fluid there are two conserved currents:

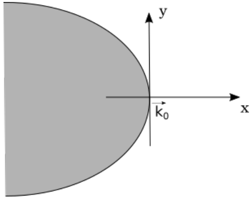
$$\partial_\mu J^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0$$

- Local thermal equilibrium: everything is function of $\mu(x)$, $T(x)$ and $u^\mu(x) \Rightarrow$ gradients expansion:

$$J^\mu = n u^\mu + \mathcal{O}(\partial), \qquad T^{\mu\nu} = (n+p) u^\mu u^\nu - p g^{\mu\nu} + \mathcal{O}(\partial)$$

Eventually one solves the EOMs order by order to find the relevant observables

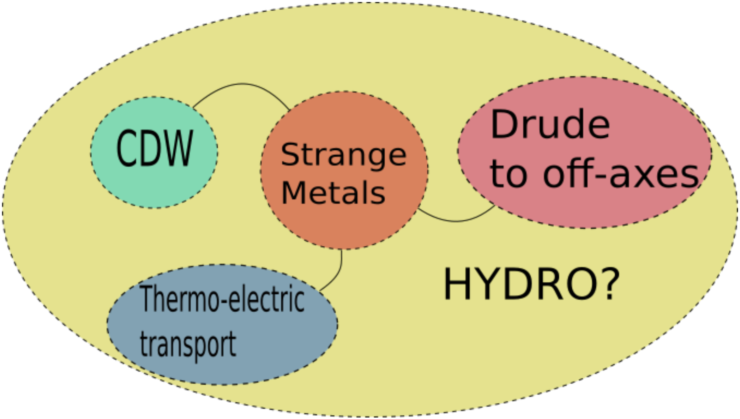
Hydrodynamics VS Fermi Liquid



- Fermi liquid has well defined quasi-particles around the Fermi Surface, which interact weakly
- To see hydrodynamics effect the interaction time must be the smallest scale in the system

Hydrodynamics is the correct EFT to describe strange metals: strongly coupled materials where the relevant long lived DOF are the (almost)-conserved currents

A unified hydrodynamic picture?



Let us play simple and start with DC transport coefficients

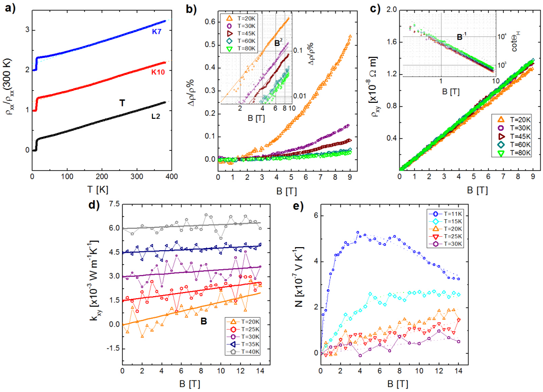
Experiment (Please be kind here!)

- We want to measure the temperature T and magnetic field B dependence of all the thermo-electric transport coefficients

- We will restrict to transverse or electric transport coefficients to avoid phonons contribution (no κ_{xx})
  - The electric conductivity ρ_{xx}
  - The Hall angle $\cot\theta_H = \frac{\rho_{xy}}{\rho_{xx}}$
  - The magnetoresistence $\frac{\rho_{xx}(B)-\rho_{xx}(0)}{\rho_{xx}(0)}$
  - The thermal Hall conductivity κ_{xy}
  - The Nernst signal N

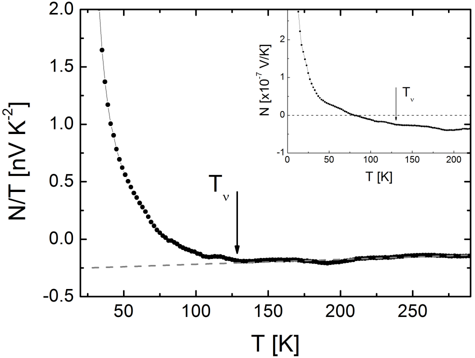
- Many coexisting phases $\Rightarrow$ we need to properly define the temperature range where the picture is supposed to be valid

B dependence of the DC transport coefficients



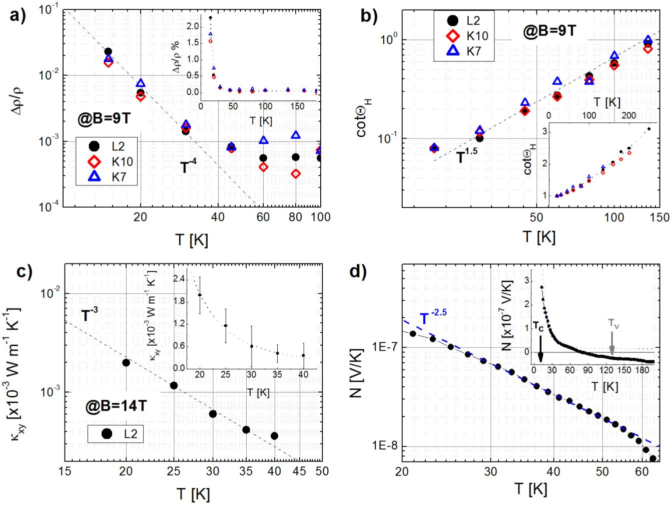
- For $T < 20$ K the Nernst starts to deviate from linearity $\Rightarrow$ Vortex effect [Wang 2006]
- For $T > 20$ K the B dependence is the one expected for a parity invariant system

T dependence of the DC transport coefficients upper bound



- **Estimation of T_ν:** the point where N/T deviates from linearity at high temperature : $T_{CDW} \sim T_\nu/2 = 65$ K [Cyr-Choinière 2009]
- In accordance with [Peng, 2018]

T dependence of the DC transport coefficients



- Relevant temperature interval 20 K $< T <$ 65 K

Summary of experimental results

- **How do experimental parameters depend on T and B?**
  - $\rho_{xx} \sim B^0 T$ as expected for strange metals.
  - $\Delta\rho/\rho \sim B^2 T^{-4}$
  - $\cot\theta_{\mathrm{H}} \sim B^{-1} T^{1.5}$ as expected in Bi-2201 but different from other materials (YBCO $\cot\theta_{\mathrm{H}} \sim B^{-1} T^2$).
  - $\kappa_{xy} \sim B T^{-3}$.
  - $N \sim B T^{-2.5}$

Hydrodynamics with broken continuous symmetries and dissipation

The breaking of translations can be pseudo-spontaneous

- Momentum dissipation rate Γ: coupling to external lattice
- phase relaxation Ω_1 of the GBs: present as soon as translations are explicitly broken [Amoretti 2018]
- The magnetic fields $F^{xy} = B$ enters only as an external field via the Lorentz term

The total EOMs:

$$\begin{aligned}
\partial_t (n, s) + \partial_i \left(J^i, Q^i/T\right) &= 0 \, , \\
\partial_t \pi^i + \partial_j T^{ji} &= F^{ij} J_j - \Gamma \pi^i - k_0^2 G \phi^i \, , \\
\partial_t \phi_a + \partial_i J^i_{\phi_a} &= -\Omega_1 \phi_a \, .
\end{aligned}$$

Constitutive relations

The only missing step is to provide constitutive relations for the currents J_i, Q_i/T, T^{ij} and $J^i_{\phi_a}$ to first order in the gradients expansion around the equilibrium configuration $T+\delta T$, $\mu+\delta\mu$:

$$
\begin{aligned}
\frac{Q^i}{T} &= s v^i - \alpha_0\left(\partial^i\delta\mu - F^{ij}v_j\right) - \frac{\bar{\kappa}_0}{T}\partial^i\delta T - \gamma_2\partial^i\theta_1\ ,\\
J^i &= n v^i - \sigma_0\left(\partial^i\delta\mu - F^{ij}v_j\right) - \alpha_0\partial^i\delta T - \gamma_1\partial^i\theta_1\ ,\\
T^{ij} &= \left(n\delta\mu + s\delta T - (G+K)\,\chi_1\theta_1\right)\delta^{ij} - G\chi_2\theta_2\epsilon^{ij}\\
&\quad -\eta\left(\partial^i v^j + \partial^j v^i - \partial_k v^k\delta^{ij}\right) - \zeta\partial_k v^k\delta^{ij} + \gamma_1 B\theta_2\delta^{ij}\ ,\\
J_1^i &= -v^i - \gamma_1\left(\partial^i\delta\mu - F^{ij}v_j\right) - \gamma_2\partial^i\delta T - \xi_1\chi_1\partial^i\theta_1 + \xi_2\chi_2\epsilon^{ij}\partial_j\theta_2\ ,\\
J_2^i &= \epsilon^{ij}J_1^j\ ,
\end{aligned}
$$

- Transport coefficients
- Susceptibilities

Constraints

- Typical constraints for charged fluid:

$$\sigma_0,\ \bar{\kappa}_0,\ \eta\ ,\Gamma\ ,\Omega_1 \geq 0\ , \qquad \bar{\kappa}_0\sigma_0 - T\alpha_0^2 \geq 0\ .$$

- Special to CDW: $\xi_1 > 0$.
- This subsequently leads to bounds on γ_1 and γ_2:

$$(\gamma_1^2, \gamma_2^2) \leq \left(\sigma_0, \frac{\bar{\kappa}_0}{T}\right) \min\left[\frac{\xi_1}{K+G}, \frac{\Omega_1}{\chi_{\pi\pi}\omega_0^2}\right]\ .$$

- We will assume $\gamma_{1,2}$ are small enough to be treated as vanishing.
- If we assume a relativistic covariant fixed point then

$$\alpha_0 = -\frac{\mu\sigma_0}{T}\ , \qquad \bar{\kappa}_0 = \frac{\mu^2\sigma_0}{T}$$

The Martin-Kadanoff method

Having the modified EOMs and the constitutive relations one can apply the Martin-Kadanoff procedure

- One can cast the EOMs in the following way (q_A are the relevant fields, s_A^0 are the sources):

$$\partial_t q_A(t, \vec{k}) + M_A^C(\vec{k}, B) s_C(t, \vec{k}) \;\; = \;\; \chi_A^B s_B^0(\vec{k}) \, .$$

- The retarded Green's function can eventually be computed

$$-\left(I_6 + i\omega \left(-i\omega I_6 + M\chi^{-1} \right)^{-1} \right) \chi \, .$$

Conductivities at low B

- Taking the DC transport coefficients to lowest order in B:
  - Charge resistivity: $\rho_{xx} = \frac{1}{\sigma_0+\tilde{\sigma}} + \mathcal{O}(B^2)$.
  - Magnetoresistance: $\frac{\Delta\rho}{\rho} = B^2 \frac{\sigma_0^3\ \tilde{\sigma}}{n^2} \frac{1}{(\sigma_0+\tilde{\sigma})^2} + \mathcal{O}(B^4)$.
  - Thermal Hall conductivity:
    $\kappa_{xy} = -BT \frac{\tilde{\sigma}^2 s}{n^4} \left(ns - 2\frac{\mu\sigma_0 n^2}{T\tilde{\sigma}} \right) + \mathcal{O}(B^3)$.
  - Hall angle: $\cot\Theta_H = \frac{n}{B\tilde{\sigma}} \frac{1+\frac{\sigma_0}{\tilde{\sigma}}}{1+2\frac{\sigma_0}{\tilde{\sigma}}} + \mathcal{O}(B)$.
  - Nernst coefficient: $N = \frac{B\ \sigma_0\ \tilde{\sigma}}{n^2(\sigma_0+\tilde{\sigma})^2}\sigma_0(s + \frac{\mu}{T}) + \mathcal{O}(B^3)$.
- DC conductivities are a sum of incoherent and relaxation conductivities

$$\sigma_{\mathrm{DC}} = \sigma_0 + \tilde{\sigma} \qquad \text{with} \qquad \tilde{\sigma} = \frac{n^2}{\chi_{\pi\pi}} \frac{\Omega_1}{\Omega_1\Gamma + \omega_0^2} \, .$$

- Only four variables σ_0, $\tilde{\sigma}$, n and s. But we measure five observables - system overconstrained.

Determining the hydrodynamic variables

- **What does experiment imply for our hydrodynamic variables?**
  - Consistency requires ρ_{xx} dominated by σ_0 at low T i.e.

$$\rho_{xx} \sim \frac{1}{\sigma_0} \sim T \,,$$

  - and

$$\cot\Theta_{\mathrm{H}} \sim \frac{n}{B\tilde{\sigma}} \sim T^{1.5} \,.$$

  - Using $\Delta\rho/\rho \sim T^{-4}$ fixes

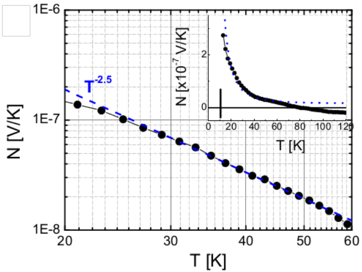
$$n \sim T^{1.5} \qquad \text{and} \qquad \tilde{\sigma} \sim T^0 \,.$$

  - Finally s is given through κ_{xy}

$$\kappa_{xy} \sim \mu B \frac{\sigma_0\ \tilde{\sigma}}{n^2} s \sim T^{-3} \qquad \Rightarrow \qquad s \sim T \,.$$

    - s is in accordance with specific heat measurement on our sample and on YBCO [Loram 1991]

Recovering the Nernst behavior



- The Nernst coefficient behaves as

$$N \sim \frac{\mu B \tilde{\sigma}}{nT} \sim \frac{\mu}{T \cot \Theta_H} \sim T^{-2.5} \ .$$

- The temperature range where the scaling agrees is exactly the one predicted from other principles (vortices at low T and $T_\nu/2$ at high T)

Outlook

- This is a consistency check of the validity of hydro
  - We can not say anything on what is dominating $\tilde{\sigma} \Rightarrow$ need for precision spectral measurements
  - If hydro is valid down to low T the Drude to off-axes peak should be explained within the same picture
- Other cuprates have different temperature scalings for the transport coefficients (eg Hall angle and κ_{xy} in YBCO)
  - CDW order is measured almost in every cuprates $\Rightarrow$ try to find a consistent picture
  - Is hydro a valid description in different point of the phase diagram?

Thank you