The emergence of the classical world from quantum theory

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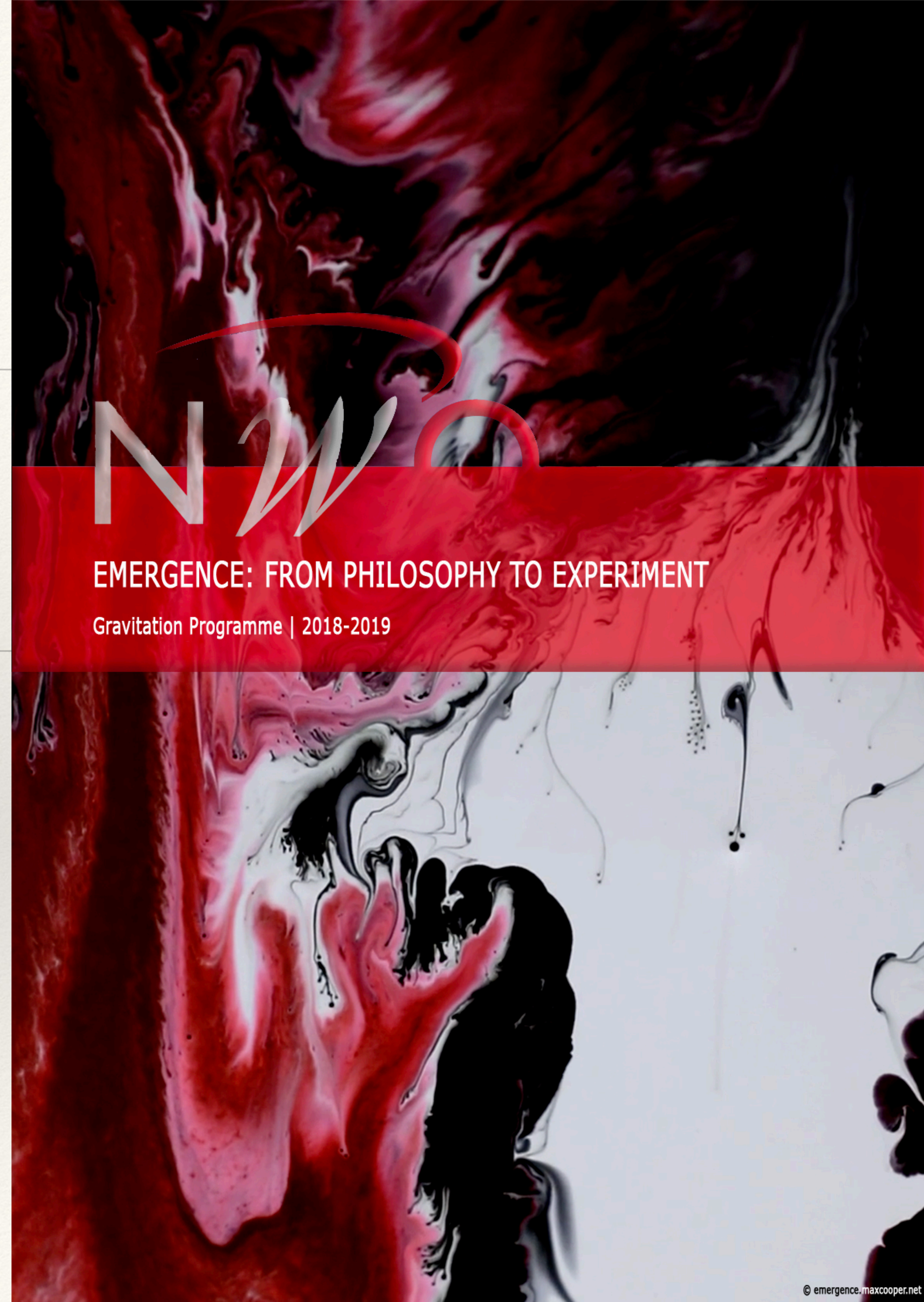


Reduction and (re)construction

- Weinberg (1995): "From Newton's time to our own we have seen a steady expansion of the range of phenomena that we know how to explain, and a steady improvement in the simplicity and universality of the theories used in these explanations. [The aim of science is] to reduce the world of physical phenomena to a finite set of fundamental equations or principles."
- Anderson (1972): "The reductionist hypothesis may still be a topic of controversy among philosophers, but among the great majority of scientists I think it is accepted without question. (…) *[But] the reductionist hypothesis does not by any means imply a 'constructionist' one:* The ability to reduce everything to fundamental laws does not imply the ability to start from those laws and reconstruct the universe. In fact, the more the elementary particle physicists tell us about the nature of the fundamental laws, the less relevance they seem to have to the very real problems of the rest of science, much less to those of society."

(Working) Definition of Emergence

- Emergence and reduction are not opposites: they *rely on each other*!
- A **set of phenomena** (usually *macroscopic and 'complex'*) has 'novel' or 'surprising' features (including laws they satisfy), which (à la Weinberg) are ontologically (or: *in principle*) reducible to **underlying constituents and their interactions** (typically both *microscopic and 'simple'*), whereas *in practice* the reconstruction of these features is so difficult that it has so far eluded us or (à la Anderson) even seems to be impossible *in practice* (so that these phenomena better be studied and funded from scratch)
- N.B. Some philosophers of mind (David Chalmers) claim that consciousness is *not* reducible to the constituents of the brain and hence is 'emergent' in a stronger sense than the one above ('strong emergence')


Emergence
H_2O molecules
Hydrodynamics
Thermodynamics
small
large

Are there any fundamental laws?

- Emergence is predicated on two well-defined different theories:
  - A *"phenomenological" theory* P (e.g. hydrodynamics)
  - A *"fundamental" theory* F (e.g. Newtonian mechanics)
- Despite this terminology a true emergentist believes that *there are no fundamental theories*: F is itself emergent (in this example, from quantum theory, where of course the buck does not stop!):

The Einstein equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$ for the gravitational field are as phenomenological (emergent from quantum theory?) as the d'Alembert equation $\partial^2\Psi/\partial t^2 - c^2\,\partial^2\Psi/\partial t^2 = 0$ for the vibrating string!

Asymptotic emergence

- “Phenomenological” theory P is *limiting case* of “fundamental” theory F: geometric optics from wave optics, hydrodynamics from molecular dynamics (double limit!), thermodynamics from (quantum) statistical mechanics, classical mechanics from QM
- Only “the right” states and observables *have* a limit (to which end one typically needs to rewrite P and F)
- This distinguishes emergence from complexity theory

Paradox of emergence

The phenomenological theory P typically has ‘emergent’ (i.e. novel, surprising, etc.) features that *seem impossible* if it is ultimately reducible to the fundamental theory F:

- ***Irreversibility*** of hydrodynamics or thermodynamics seems *incompatible* with reversibility of equations of motion in (Newtonian) molecular dynamics
- ***Spontaneous Symmetry Breaking*** in thermodynamics seems *incompatible* with quantum statistical mechanics (almost any finite quantum system has a unique ground state and any finite quantum system has a unique equilibrium state)
- ***Measurement outcomes*** seem *incompatible* with unitary quantum theory (hence von Neumann’s “collapse of the wave-function” must be *imposed* on QM)

Nature (being reversible, finite, quantum) *should* be described by F rather than P, but *in fact* has the features predicted by P (irreversibility, SSB, measurement outcomes)

Resolution of paradox of emergence

For asymptotic emergence this problem is captured by ***Earman's Principle*** (2004):

- *"While idealizations* [inherent in using the phenomenological theory P] *are useful and, perhaps, even essential to progress in physics, a sound principle of interpretation would seem to be that no effect can be counted as a genuine physical effect if it disappears when the idealizations are removed"* [i.e. falling back to the fundamental theory F].

One can only satisfy this principle *and* have emergence if, against appearances, F has properties *foreshadowing* the seemingly paradoxical emergent properties of P:

- ***Butterfield's Principle*** (2011), the contrapositive to Earman's Principle, states this: *"there is a weaker, yet still vivid, novel and robust behaviour that occurs **before** we get to the limit, i.e. for finite N. And it is this weaker behaviour which is physically real."*

F's role towards the emergent theory P unearths new properties of F, so novel and surprising properties of P already originate in novel and surprising properties of F

Classical limit of QM

$\hbar \to 0$ limit of QM and $N \to \infty$ limit of quantum statistical mechanics

WELL UNDERSTOOD

- Converge of mathematical structures (Hilbert space → phase space, operators → functions on phase space)
- Convergence of partition functions, coherent states, WKB wave-functions, etc.
- Convergence of dynamics ("Ehrenfest's Theorem")

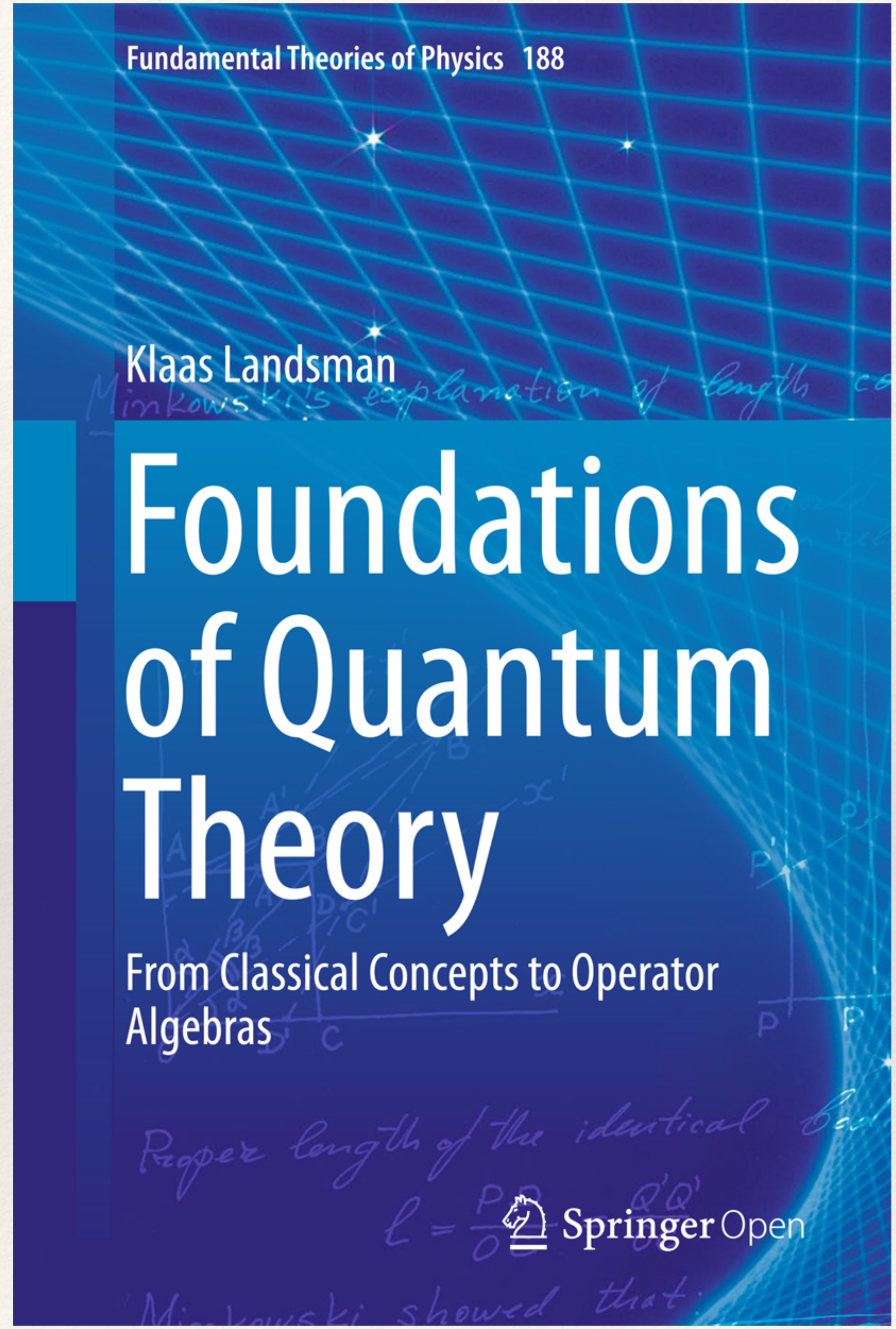
NOT UNDERSTOOD

- Schrödinger Cat states (which do have a classical limit but the "wrong" one: mixed, not pure)
- SSB (Cat with multiple states and equal Born probabilities)

*All problems with the classical limit reflect (traditional) problems in the foundations of quantum theory: notably the **measurement problem***

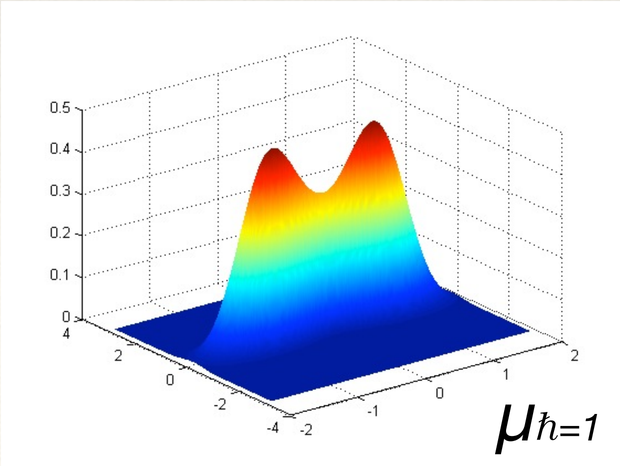
WELL UNDERSTOOD (1998)

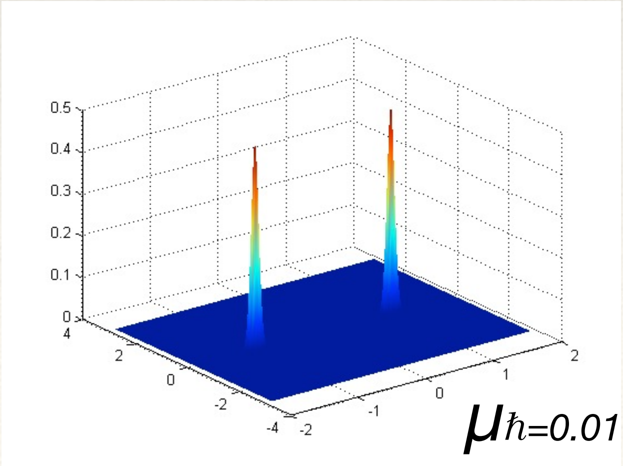
NOT UNDERSTOOD (2017)

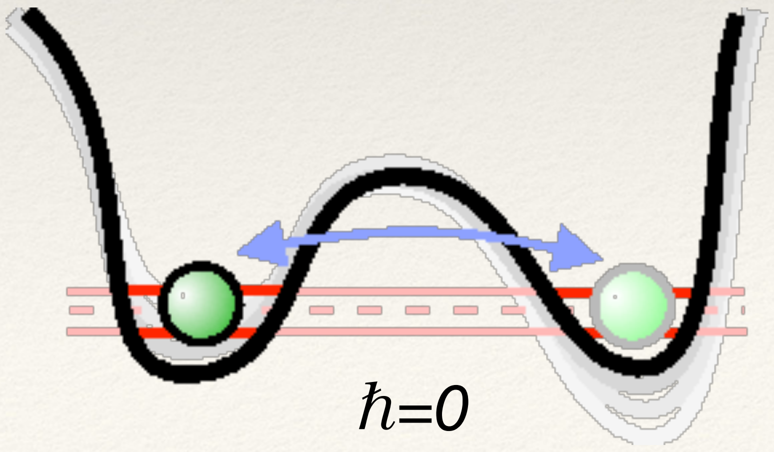


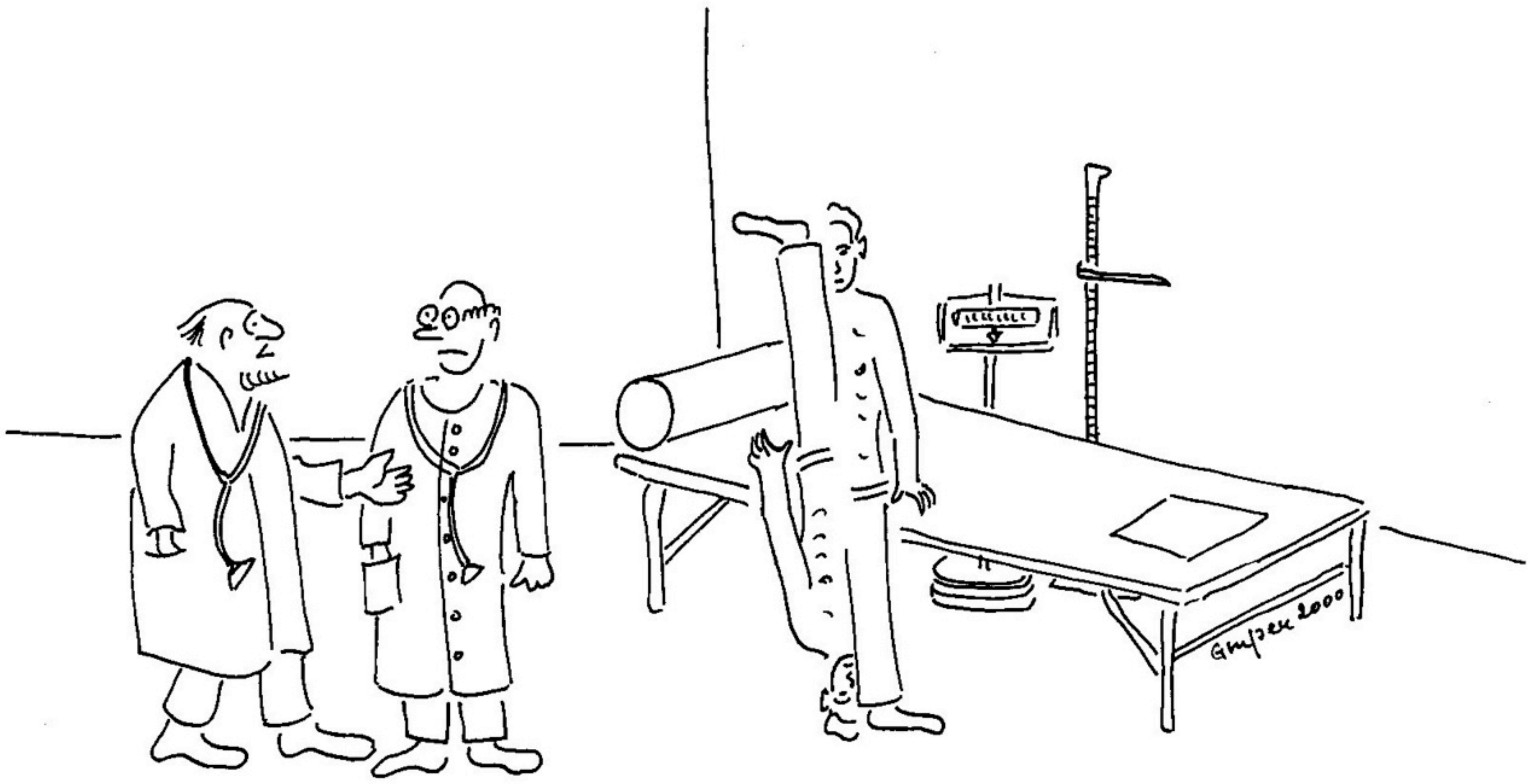
Case study: SSB in CM and QM

- Particle moving in double well potential in $d = 1$
- Hamiltonian has $\mathbb{Z}_2$ (reflection) symmetry: $h(p,q) = h(p, -q)$
- ***Classical*** (pure) ground states: $(p_0 = 0, q_0 = \pm a) \in \mathbb{R}^2$: **SSB**
- Either *asymmetric pure* states or *symmetric mixed* state
- ***Quantum*** ground state is unique (**NO SSB**) and (seen as Husimi function) its classical limit is mixed state $\mu_0 = \frac{1}{2}(\delta_+ + \delta_-)$





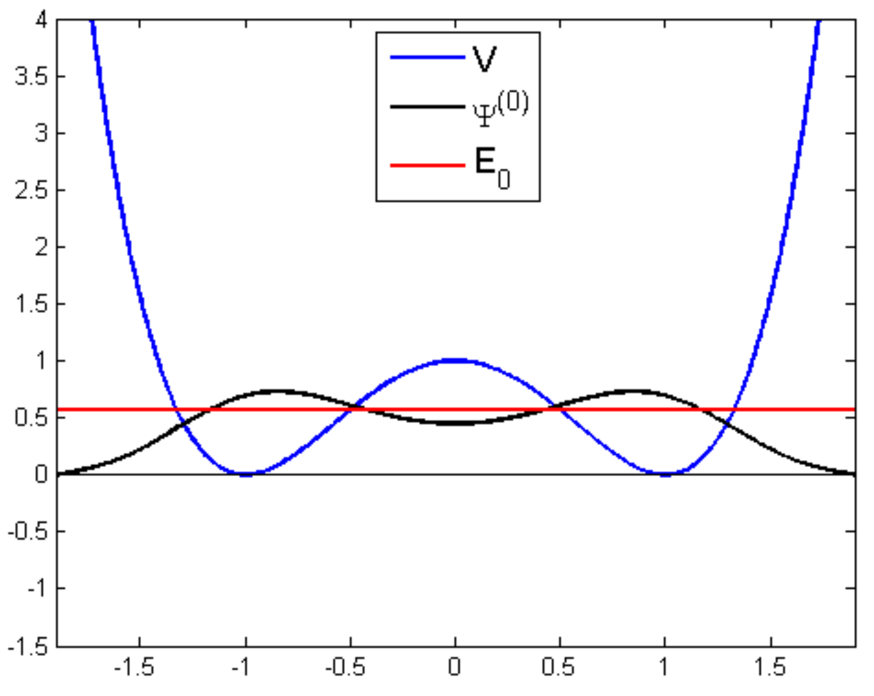




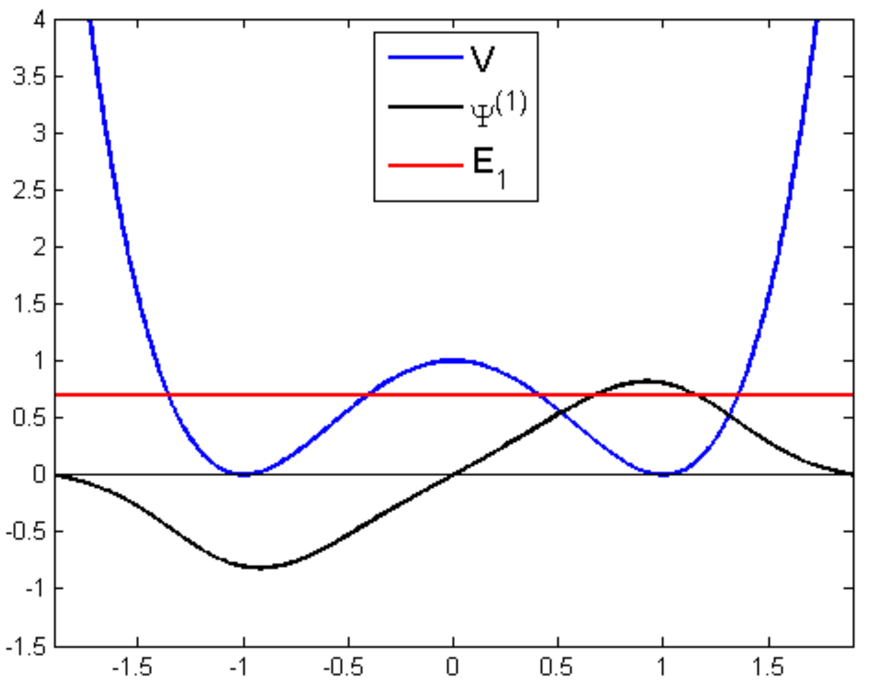
"A severe case of symmetry breaking!"

What, then, are the "novel and surprising properties" of the fundamental theories F = QM or F = QSM that give rise to the "novel and surprising property" of SSB in the phenomenological theories P = classical mechanics or P = thermodynamics?

First excited state in double well potential



ground state



first excited state

- Energy difference $\Delta E = E_1 - E_0 \sim \hbar \exp\left(-\frac{1}{\hbar}\int_{-a}^{a} dx\,\sqrt{V(x)}\right)$

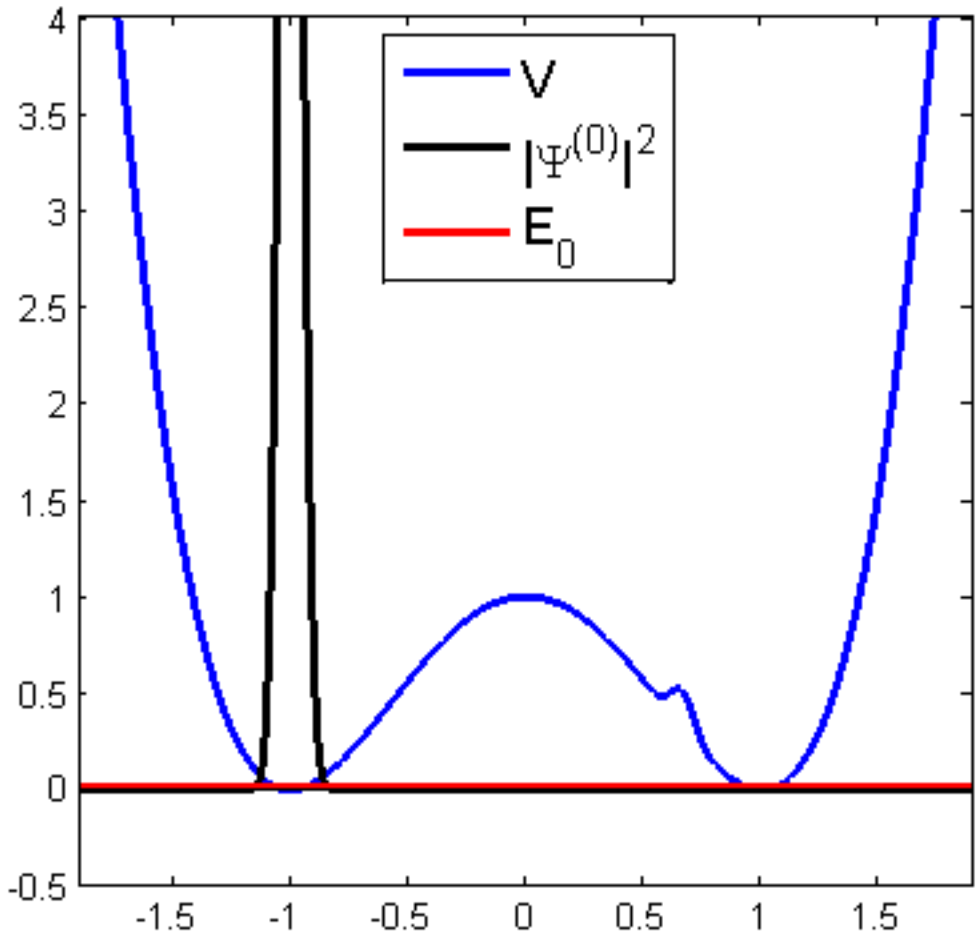
- $\Psi_\hbar^{(\pm)} = (\Psi_\hbar^{(0)} \pm \Psi_\hbar^{(1)})/\sqrt{2}$ are ***localized*** (singly peaked) states, up to *exp*(-1/*ħ)*

Converge to (pointwise) localized pure (*physical*) classical ground states $\mu_\pm$

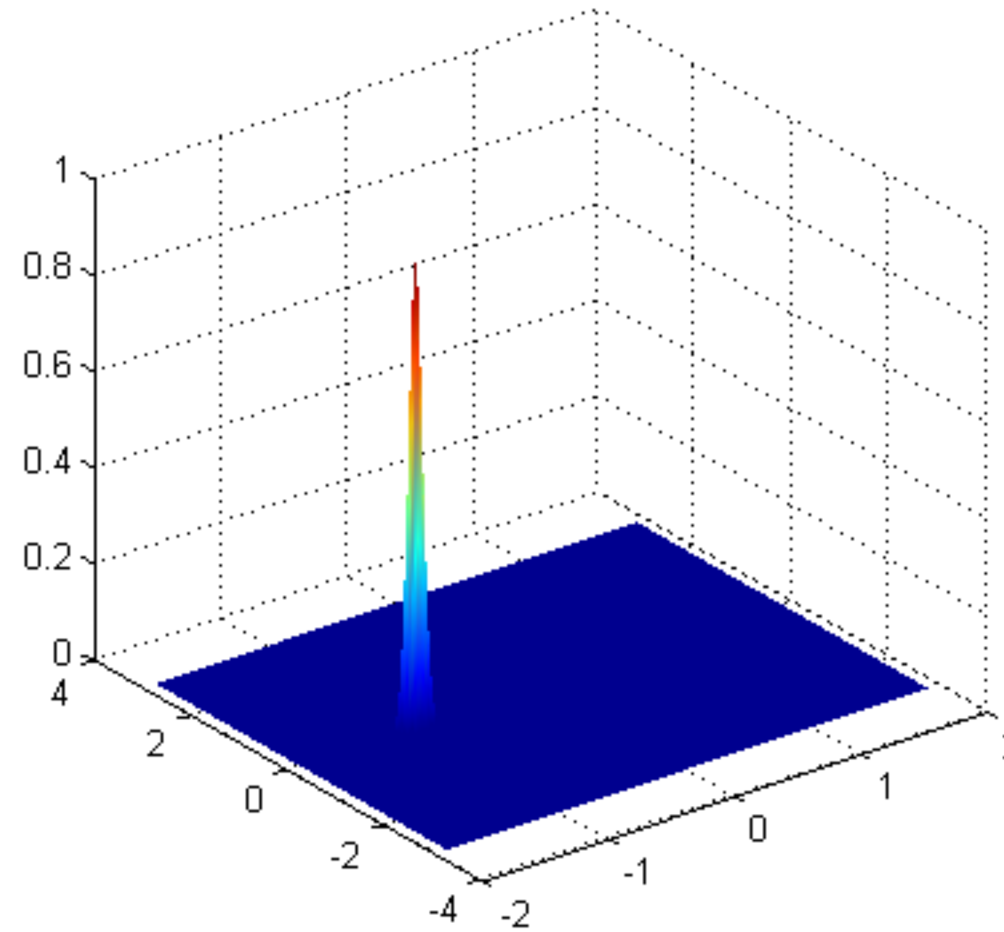
General picture

- In all known (non gauge) cases of SSB (also for continuous symmetries), low-lying states (infinitely many for continuous symmetries) of a **large** system can form linear combinations, having almost the ground state energy, whose thermodynamic limits are the degenerate symmetry-breaking ground states of the corresponding **infinite** system (Koma & Tasaki, 1994)
- This is the first step towards resolving the paradox of emergence; the second step is:

The flea on Schrödinger's Cat



Ground state for perturbed DW potential



Husimi function for $\hbar = 0.01$

These symmetry-breaking combinations of low-lying states are (almost exactly) ground states of the arbitrarily small asymmetrically **perturbed** potential (*"flea" perturbation"*) (Jona-Lasinio, Martinelli, & Scoppola, $\hbar \to 0$, *1981*, van Wezel, Zaanen, $N \to \infty$, *2005-6*)

Summary and conclusion

- Classical mechanics (or thermodynamics) arises from QM (or quantum statistical mechnaics) through *asymptotic emergence*, i.e. as a "phenomenological theory" P that is: (i) a limiting case of a "fundamental" theory F, and (ii) has surprising and novel features (such as SSB) that even seem *forbidden* by F
- This is highly paradoxical since the actual (QM) world should be described by F and only *appears* to behave according to P —similarly for all other cases: the (hydrodynamic, thermodynamic, classical …) limit is not reached in Nature
- To overcome this "paradox of emergence", theory F must *itself* already have surprising and novel features *that only come to light in its asymptotic relation to P*
- Towards SSB these features include extreme sensitivity to perturbations in the pertinent limit ($\hbar \to 0$ in QM and $N \to \infty$ *in* quantum statistical mechanics)